Osnovne formule iz Analize II i Analize III

Dio tablica integrala.

1. $\int u^a \mathrm{d}u = \dfrac{u^{a+1}}{a+1} + C,\ a \neq -1.$
2. $\int u^{-1} \mathrm{d}u = \int \dfrac{\mathrm{d}u}{u} = \int \dfrac{u'}{u} \mathrm{d}x = \ln|u| + C.$
3. $\int a^u \mathrm{d}u = \dfrac{a^u}{\ln|a|} + C; \int e^u \mathrm{d}u = e^u + C.$
4. $\int \sin \mathrm{d}u = -\cos u + C.$
5. $\int \cos \mathrm{d}u = \sin u + C.$
6. $\int \dfrac{1}{\cos^2 u} \mathrm{d}u = \operatorname{tg} u + C.$
7. $\int \dfrac{1}{\sin^2 u} \mathrm{d}u = -\operatorname{ctg} u + C.$
8. $\int \dfrac{\mathrm{d}u}{u^2 + a^2} = \dfrac{1}{a} \operatorname{arc\,tg} \dfrac{u}{a} + C.$
9. $\int \dfrac{\mathrm{d}u}{u^2 - a^2} = \dfrac{1}{2a} \ln\left|\dfrac{u-a}{u+a}\right| + C.$
10. $\int \dfrac{\mathrm{d}u}{\sqrt{a^2 - u^2}} = \arcsin \dfrac{u}{a} + C.$
11. $\int \dfrac{\mathrm{d}u}{\sqrt{u^2 + a}} = \ln|u + \sqrt{u^2 + a}| + C.$

Newton-Leibnizova formula.

$$\int_a^b f(u)\mathrm{d}u = \int f(u)\mathrm{d}u\Big|_a^b = F(u)|_a^b = F(b) - F(a), \text{ gdje je } F'(u) = f(u).$$

Osobine određenih integrala.

1. $\int_a^b f(x)\mathrm{d}x = -\int_b^a f(x)\mathrm{d}x.$
2. $\int_a^a f(x)\mathrm{d}x = 0.$
3. $\int_a^b f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x + \int_c^b f(x)\mathrm{d}x.$
4. $\int_a^b [f_1(x) + f_2(x) - f_3(x)]\mathrm{d}x = \int_a^b f_1(x)\mathrm{d}x + \int_a^b f_2(x)\mathrm{d}x - \int_a^b f_3(x)\mathrm{d}x.$
5. $\int_a^b c\, f(x)\mathrm{d}x = c \int_a^b f(x)\mathrm{d}x.$

Smjena promjenjivih u određenom integralu.

$$\int_a^b f(x)\mathrm{d}x = \left| \begin{array}{ll} x = \varphi(t) & x = a \Rightarrow a = \varphi(\alpha) \Rightarrow t = \alpha \\ dx = \varphi'(t)\mathrm{d}t & x = b \Rightarrow b = \varphi(\beta) \Rightarrow t = \beta \end{array} \right| = \int_\alpha^\beta f(\varphi(t))\varphi'(t)\mathrm{d}t = \int_\alpha^\beta h(t)\mathrm{d}t$$

Nepravi integrali. $\displaystyle\int_a^{+\infty} f(x)\mathrm{d}x = \lim_{b\to\infty} \int_a^b f(x)\mathrm{d}x, \int_{-\infty}^{b} f(x)\mathrm{d}x = \lim_{a\to-\infty} \int_a^b f(x)\mathrm{d}x,...$

Računanje površine ravne figure. U zavisnosti od izgleda slike: $P = \int_a^b f(x)\mathrm{d}x$, $P = \int_c^d g(y)\mathrm{d}y$, $P = -\int_a^b f(x)\mathrm{d}x$, $P = \int_a^b [\eta(x) - \mu(x)]\mathrm{d}x$, $P = \int_c^d [g(y) - h(y)]\mathrm{d}y$, ...

Zapremina rotacionog tijela. Ako, kriva data u parametarskom obliku $C : \begin{cases} x = \eta(t) \\ y = \mu(t) \\ t_1 \leq t \leq t_2 \end{cases}$ rotira oko x-ose, zapremine se računa po formuli

$$V_x = \pi \int_{t_1}^{t_2} [\mu(t)]^2 |\eta'(t)| \mathrm{d}t.$$

Ista kriva ako rotira oko y-ose, $V_y = \pi \int_{t_1}^{t_2} [\eta(t)]^2 |\mu'(t)| \mathrm{d}t$. Iz ove dvije formule, za funkcije $y = f(x)$ i $x = g(y)$, slijedi $V_x = \pi \int_a^b [f(x)]^2 \mathrm{d}x$ i $V_y = \pi \int_c^d [g(y)]^2 \mathrm{d}y$.

Dužina luka krive. $C:\begin{cases} x=\eta(t)\\ y=\mu(t)\\ t_1\le t\le t_2\end{cases}$, $\ell=\int\limits_{t_1}^{t_2}\sqrt{[\eta'(t)]^2+[\mu'(t)]^2}\mathrm{d}t$;

$C:\begin{cases} y=f(x)\\ a\le x\le b\end{cases}$, $\ell=\int\limits_a^b\sqrt{1+[y'(x)]^2}\mathrm{d}x$; $C:\begin{cases} x=g(y)\\ c\le y\le d\end{cases}$, $\ell=\int\limits_c^d\sqrt{1+[g'(y)]^2}\mathrm{d}y$;

Komplanacija obrtne površi. Površina omotača tijela dobijenog rotacijom krive $C:\begin{cases} x=\eta(t)\\ y=\mu(t)\\ t_1\le t\le t_2\end{cases}$, oko x-ose, se računa po formuli: $P=2\pi\int\limits_{t_1}^{t_2}|\mu(t)|\sqrt{[\eta'(t)]^2+[\mu'(t)]^2}\mathrm{d}t$;

$C:\begin{cases} y=f(x)\\ a\le x\le b\end{cases}$, $P=2\pi\int\limits_a^b|f(x)|\sqrt{1+[f'(x)]^2}\mathrm{d}x$; ...

Razvijanje funkcije u Furijerov red na intervalu $[a,b]$, $a<b$.
$a_0=\frac{2}{b-a}\int_a^b f(x)\mathrm{d}x$, $a_n=\frac{2}{b-a}\int_a^b f(x)\cos\frac{2n\pi x}{b-a}\mathrm{d}x$, $b_n=\frac{2}{b-a}\int_a^b f(x)\sin\frac{2n\pi x}{b-a}\mathrm{d}x$,

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi x}{b-a}+b_n\sin\frac{2n\pi x}{b-a}\right),\ \ x\in[a,b]$$

Funkcije dvije nazavisno promjenjive. Neka je S neprazan skup prostora $\mathbb{R}^2$ i neka je $T\subseteq\mathbb{R}$. Ako svakoj tački $(x;y)\in S$ možemo unaprijed po datom pravilu f pridružiti jednu i samo jednu realnu vrijednost $z\in T$, tada kažemo da je data realna funkcija dvije realne promjenjive f iz $\mathbb{R}^2$ u $\mathbb{R}$ (sa skupa $S\subseteq\mathbb{R}^2$ u skup $T\subseteq\mathbb{R}$) i pišemo $z=f(x,y)$.

Limesi i neprekidnost funkcija dvije nezavisno promjenjive. Za funkciju f dvije promjenjive kažemo da je neprekidna u tački (a,b) akko

$$\lim_{(x,y)\to(a,b)}f(x,y)=f(a,b).$$

Kažemo da je f neprekidna na oblasti D ako je neprekidna u svakoj tački $(a,b)\in D$.

Parcijalni izvodi f-ja više pomjenjivih. $z=f(x,y)$, $z'_x=\frac{\partial z}{\partial x}=\lim\limits_{\Delta x\to0}\frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$ ...

Diferenciranje funkcija više promjenjivih. $u=f(x,y,z)$, $du=\frac{\partial u}{\partial x}\mathrm{d}x+\frac{\partial u}{\partial y}\mathrm{d}y+\frac{\partial u}{\partial z}\mathrm{d}z$ ...

Diferenciranje složenih funkcija. Neka je $z=F(u,v,w)$ gdje je $u=f(x,y,t)$, $v=\varphi(x,y,t)$ i $w=\psi(x,y,t)$. Tada

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}+\frac{\partial z}{\partial w}\cdot\frac{\partial w}{\partial x}$$
$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}+\frac{\partial z}{\partial w}\cdot\frac{\partial w}{\partial y}$$
$$\frac{\partial z}{\partial t}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial t}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial t}+\frac{\partial z}{\partial w}\cdot\frac{\partial w}{\partial t}$$

Ako je $z=F(u,v,w)$ gdje je $u=f(x)$, $v=\varphi(x)$ i $w=\psi(x)$. Tada

$$\frac{\mathrm{d}z}{\mathrm{d}x}=\frac{\partial z}{\partial u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{\partial z}{\partial v}\cdot\frac{\mathrm{d}v}{\mathrm{d}x}+\frac{\partial z}{\partial w}\cdot\frac{\mathrm{d}w}{\mathrm{d}x}$$

Parcijalni izvodi višeg reda složenih funkcija.

$$\frac{\partial^2 z}{\partial x^2}=f''_{xx}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial^2 z}{\partial x\partial y}=f''_{xy}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right),\quad \mathrm{d}^2z=\frac{\partial^2 z}{\partial x^2}\mathrm{d}x^2+2\frac{\partial^2 z}{\partial x\partial y}\mathrm{d}x\mathrm{d}y+\frac{\partial^2 z}{\partial y^2}\mathrm{d}y^2$$

Tejlorov red za funkciju dvije i više promjenjivih.

$$f(x,y) \sim f(p_1,p_2) + \sum_{n=1}^{\infty} \frac{1}{n!} \left((x-p_1)\frac{\partial}{\partial x} + (y-p_2)\frac{\partial}{\partial y} \right)^h f(p_1,p_2)$$

Jednačina tangentne ravni i jednačina normale na površ. Ako površ S ima jednačinu u implicitnom obliku $F(x,y,z)=0$

$$\alpha : F'_x(p_1,p_2,p_3)(x-p_1) + F'_y(p_1,p_2,p_3)(y-p_2) + F'_z(p_1,p_2,p_3)(z-p_3) = 0$$

$$n : \frac{x-p_1}{F'_x(p_1,p_2,p_3)} = \frac{y-p_2}{F'_y(p_1,p_2,p_3)} = \frac{z-p_3}{F'_z(p_1,p_2,p_3)}$$

Izvod funkcije u datom smjeru. Gradijent funkcije. Za datu funkciju $u = f(x,y,z)$

$$\mathrm{grad}u(M) = \left(\frac{\partial u}{\partial x}(M), \frac{\partial u}{\partial y}(M), \frac{\partial u}{\partial z}(M) \right), \quad \frac{\partial u}{\partial \vec{e}}(M) = \mathrm{grad}u(M) \cdot \vec{e}$$

Ekstremne vrijednosti f-ja dvije promjenjive. $z = f(x,y)$ 1. $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, 2. rješavamo sistem $\frac{\partial z}{\partial x} = 0$, $\frac{\partial z}{\partial y} = 0$ 3. rješenjem sistema dobijamo parcijalne tačke $M_1(x_1;y_1)$, $M_2(x_2;y_2)$, ..., $M_n(x_n;y_n)$, 4. $A = \frac{\partial^2 z}{\partial x^2}(M_k)$, $B = \frac{\partial^2 z}{\partial x \partial x}(M_k)$, $C = \frac{\partial^2 z}{\partial y^2}(M_k)$, $D = AC - B^2$, ...

Ekstremne vrijednosti f-ja tri promjenjive. ...

Uslovni ekstremi f-ja dvije promjenjive. ...

Dvojni integrali.

$$\iint\limits_D f(x,y) = \int\limits_a^b \mathrm{d}x \int\limits_{g(x)}^{h(x)} f(x,y)\mathrm{d}y = \int\limits_a^b \left[\int\limits_{g(x)}^{h(x)} f(x,y)\mathrm{d}y \right] \mathrm{d}x,$$

$$\iint\limits_D f(x,y) = \int\limits_c^d \mathrm{d}y \int\limits_{\eta(y)}^{\mu(y)} f(x,y)\mathrm{d}x = \int\limits_c^d \left[\int\limits_{\eta(y)}^{\mu(y)} f(x,y)\mathrm{d}x \right] \mathrm{d}y \ldots$$

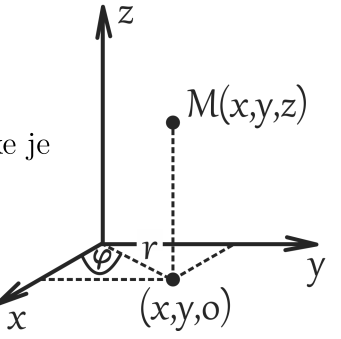
Smjena promjenjivih u dvojnim integralima. Za prelazak sa pravougaonih na polarne koordinate koristimo smjene $\begin{cases} x = r\cos(\varphi) \\ y = r\sin(\varphi) \\ \mathrm{d}x\mathrm{d}y = r\mathrm{d}r\mathrm{d}\varphi \end{cases}$, poopštene plarne koordinate su oblika $\begin{cases} x = a\,r\cos(\varphi),\ (a>0) \\ y = b\,r\sin(\varphi),\ (b>0) \\ \mathrm{d}x\mathrm{d}y = a\,b\,r\mathrm{d}r\mathrm{d}\varphi \end{cases}$, a za proizvoljne smjene $\begin{cases} x = \eta(u,v) \\ y = \mu(u,v) \\ \mathrm{d}x\mathrm{d}y = |J|\mathrm{d}u\mathrm{d}v \end{cases}$, gdje je J Jakobijan,

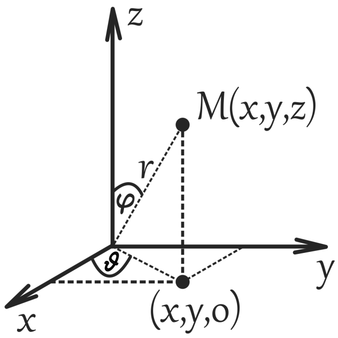
$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Trojni integrali. ...

Računanje trojnih integrala uvođenjem cilindričnih i sfernih koordinata.

Na cilindrične koordinate prelazimo pomoću $\begin{cases} x = r\cos(\varphi) \\ y = r\sin(\varphi) \\ z = z \\ \mathrm{d}x\mathrm{d}y\mathrm{d}z = r\mathrm{d}r\mathrm{d}\varphi\mathrm{d}z \end{cases}$, opis tačke je





Za prelazak sa pravougaonih na sferne koordinate koristimo sljedeće smjene $\begin{cases} x = r\sin(\varphi)\cos(\theta) \\ y = r\sin(\varphi)\sin(\theta) \\ z = r\cos(\varphi) \\ \mathrm{d}x\mathrm{d}y\mathrm{d}z = r^2\sin(\varphi)\mathrm{d}r\mathrm{d}\varphi\mathrm{d}\theta \end{cases}$, (opis tačke je prikazan na slici lijevo).

Primjena dvostrukih integrala.

$$(a)\ P = \iint\limits_D \mathrm{d}x\mathrm{d}y. \qquad (b)\ V = \iint\limits_D f(x,y)\mathrm{d}x\mathrm{d}y.$$

Primjena trostrukih integrala. $(a)\ V = \iiint\limits_\Omega \mathrm{d}x\mathrm{d}y\mathrm{d}z.$

$$(b)\ T(x_T, y_T, z_T),\ \ x_T = \frac{1}{V}\iiint\limits_\Omega x\mathrm{d}x\mathrm{d}y\mathrm{d}z, y_T = \frac{1}{V}\iiint\limits_\Omega y\mathrm{d}x\mathrm{d}y\mathrm{d}z, z_T = \frac{1}{V}\iiint\limits_\Omega z\mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

Krivoliniski integral prve vrste (po luku).

$$C: \begin{cases} x = \eta(t) \\ y = \mu(t) \\ t_1 \le t \le t_2 \end{cases}, \qquad \int\limits_C f(x,y)\mathrm{d}s = \int\limits_{t_1}^{t_2} f(\eta(t),\mu(t))\sqrt{(\eta'(t))^2 + (\mu'(t))^2}\mathrm{d}t.$$

$$C: \begin{cases} y = f(x) \\ a \le x \le b \end{cases}, \qquad \int\limits_C z(x,y)\mathrm{d}s = \int\limits_a^b z(x,f(x))\sqrt{1 + (f'(x))^2}\mathrm{d}x.$$

Primjena krivoliniskog integrala prve vrste - Računanje površine cilindrične površi.

$$C: \begin{cases} F(x,y) = 0 \\ z = 0 \end{cases}, \qquad P = \int\limits_C z(x,y)\mathrm{d}s.$$

Krivoliniski integral druge vrste (po koordinatama).

$$C: \begin{cases} x = \eta(t) \\ y = \mu(t) \\ t_1 \le t \le t_2 \end{cases}, \qquad \int\limits_C P(x,y)\mathrm{d}x + Q(x,y)\mathrm{d}y = \int\limits_{t_1}^{t_2} [P(\eta(t),\mu(t))\eta'(t) + Q(\eta(t),\mu(t))\mu'(t)]\mathrm{d}t.$$

$$C: \begin{cases} y = f(x) \\ a \le x \le b \end{cases}, \qquad \int\limits_C P(x,y)\mathrm{d}x + Q(x,y)\mathrm{d}y = \int\limits_a^b [P(x,f(x)) + Q(x,f(x))f'(x)]\mathrm{d}x.$$

Krivoliniski integral druge vrste <u>*ovisi o smjeru puta integracije*</u>.

Formula Greena.

$$\int\limits_C P(x,y)\mathrm{d}x + Q(x,y)\mathrm{d}y = \iint\limits_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathrm{d}x\mathrm{d}y.$$

Primjena krivoliniskog integrala druge vrste - Računanje površine ravne figure.

$$P = \frac{1}{2}\int\limits_C x\mathrm{d}y - y\mathrm{d}x.$$

Nezavisnost krivoliniskog integrala od vrste konture. Određivanje primitivnih funkcija.

..., $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$, ..., $\mathrm{d}u(x,y) = \dfrac{\partial u}{\partial x}\mathrm{d}x + \dfrac{\partial u}{\partial y}\mathrm{d}y$, ...

Površinski integral prve vrste.

D projekcija od S: $z=\eta(x,y)$ na $x0y$ - $\iint\limits_S f(x,y,z)\mathrm{d}S=\iint\limits_D f(x,y,\eta(x,y))\sqrt{1+\left(\frac{\partial\eta}{\partial x}\right)^2+\left(\frac{\partial\eta}{\partial y}\right)^2}\mathrm{d}x\mathrm{d}y.$

E projekcija od S: $y=\mu(x,z)$ na $x0z$ - $\iint\limits_S f(x,y,z)\mathrm{d}S=\iint\limits_E f(x,\mu(x,z),z)\sqrt{1+\left(\frac{\partial\mu}{\partial x}\right)^2+\left(\frac{\partial\mu}{\partial z}\right)^2}\mathrm{d}x\mathrm{d}z.$

F projekcija od S: $x=\gamma(y,z)$ na $y0z$ - $\iint\limits_S f(x,y,z)\mathrm{d}S=\iint\limits_F f(\gamma(y,z),y,z)\sqrt{1+\left(\frac{\partial\gamma}{\partial y}\right)^2+\left(\frac{\partial\gamma}{\partial z}\right)^2}\mathrm{d}y\mathrm{d}z.$

Površinski integral druge vrste.
Ako je integral oblika $\iint\limits_S P(x,y,z)\mathrm{d}y\mathrm{d}z+Q(x,y,z)\mathrm{d}x\mathrm{d}z+R(x,y,z)\mathrm{d}x\mathrm{d}y$ obično ga podjelimo na tri dijela $\iint\limits_S P(x,y,z)\mathrm{d}y\mathrm{d}z$, $\iint\limits_S Q(x,y,z)\mathrm{d}x\mathrm{d}z$, $\iint\limits_S R(x,y,z)\mathrm{d}x\mathrm{d}y$. Neka je $\vec{n}=(\cos\alpha,\cos\beta,\cos\gamma)$ vektor normale na površinu S, gdje su α, β i γ uglovi koje vektor normale zaklapa sa x, y i z osom. Tada

$$I_1=\iint\limits_S P(x,y,z)\mathrm{d}y\mathrm{d}z=\left|\begin{array}{c} S: x=\eta(y,z),\\ \text{neka je } D \text{ projekcija od } S \text{ na } y0z \text{ ravan,}\\ \text{neka je } \alpha \text{ ugao koji vektor}\\ \text{normale na } S \text{ zaklapa sa } x\text{-osom,}\end{array}\right|=\pm\iint\limits_D P(\eta(y,z),y,z)\mathrm{d}y\mathrm{d}z \text{ gdje}$$

vrijednost za $\pm$ zavisi od $\cos(\alpha)$ ($\cos(\alpha)>0$ stavljamo +, za $\cos(\alpha)<0$ stavljamo -, a za $\cos(\alpha)=0$ imamo $I_1=0$). Slično za I_2 i I_3

$$I_2=\iint\limits_S Q(x,y,z)\mathrm{d}x\mathrm{d}z=\left|\begin{array}{c} S: y=\mu(x,z),\\ \text{neka je } E \text{ projekcija od } S \text{ na } x0z \text{ ravan,}\\ \text{neka je } \beta \text{ ugao koji vektor}\\ \text{normale na } S \text{ zaklapa sa } y\text{-osom,}\end{array}\right|=\pm\iint\limits_E Q(x,\mu(x,z),z)\mathrm{d}x\mathrm{d}z.$$

$$I_3=\iint\limits_S R(x,y,z)\mathrm{d}x\mathrm{d}y=\left|\begin{array}{c} S: z=\delta(x,y),\\ \text{neka je } F \text{ projekcija od } S \text{ na } x0y \text{ ravan,}\\ \text{neka je } \gamma \text{ ugao koji vektor}\\ \text{normale na } S \text{ zaklapa sa } x\text{-osom,}\end{array}\right|=\pm\iint\limits_F R(x,y,\delta(x,y))\mathrm{d}x\mathrm{d}y.$$

Primjena površinskog integrala prve vrste - Izračunavanje površine dijela glatke površi.

$P=\iint\limits_S \mathrm{d}S=\iint\limits_D \sqrt{1+\left(\frac{\partial\eta}{\partial x}\right)^2+\left(\frac{\partial\eta}{\partial y}\right)^2}\mathrm{d}x\mathrm{d}y$, gdje je D projekcija od S: $z=\eta(x,y)$ na $x0y$ ravan.

Stoksova formula. ...

Formula Gauss-Ostrogradski.

$$\iint\limits_S P(x,y,z)\mathrm{d}y\mathrm{d}z+Q(x,y,z)\mathrm{d}x\mathrm{d}z+R(x,y,z)\mathrm{d}x\mathrm{d}y=\iiint\limits_\Omega \left(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

Integrali ovisni o parametru.

$I(\alpha)=\int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\mathrm{d}x \quad\Longrightarrow\quad I'(\alpha)=\int_{a(\alpha)}^{b(\alpha)} f'_\alpha(x,\alpha)\mathrm{d}x+b'(\alpha)f(b(\alpha),\alpha)-a'(\alpha)f(a(\alpha),\alpha).$

Ako granice a i b ne zavise od α tada $I'(\alpha)=\int\limits_a^b f'_\alpha(x,\alpha)\mathrm{d}x.$

Vektorska teorija polja. ...

Cirkulacija i fluks vektorskog polja.

$$C=\int\limits_c \vec{v}\,\mathrm{d}\vec{r}=\int\limits_c v_x\mathrm{d}x+v_y\mathrm{d}y+v_z\mathrm{d}z.$$

$$\Phi=\iint\limits_S \vec{v}\vec{n}\,\mathrm{d}S=\iint\limits_S v_x\mathrm{d}y\mathrm{d}z+v_y\mathrm{d}x\mathrm{d}z+v_z\mathrm{d}x\mathrm{d}z.$$